Roll No. ........................

Total No. of Questions : 09] [Total Pages : 05

(2125)

# 2504473

## M.A./M.Sc. (CBCS)

## EXAMINATION, 2025

(Third Semester)

MATHEMATICS

M-301

Complex Analysis

*Time : 3 Hours*] [*Maximum Marks* : 80

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The candidates shall limit their answer precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

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**Note :** Attempt *five* questions in all, selecting at least *one* question (but not more than two questions) from each Section. Each question will be of 16 marks.

## Section A

1. (a) State and prove Cauchy's inequality in complex numbers. **8**

   (b) Prove that $\left|\dfrac{a-b}{1-b\overline{a}}\right| <$ if $|a| < 1$ and $|b| < 1$.

   **8**

2. (a) State and prove Lucas' theorem. **8**

   (b) Show that an analytic function cannot have a constant absolute value without reducing to a constant. **8**

3. (a) Show that all circles that pass through $a$ and $\dfrac{1}{\overline{a}}$ intersect the circle $|z| = 1$ at right angles. **8**

   (b) If $f(z)$ is an analytic function of $z$, prove that $\left(\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}\right)|\operatorname{Re} f(z)|^2 = 2|f'(z)|^2$.

   **8**

$v_x =$ $u_x$ $v_y =$

## Section B

**4.** (a) State and prove Cauchy's theorem for a rectangle.

(b) Find the radius of convergence of the following power series :

(i) $\sum_0^{\infty} \dfrac{z^n}{2^n+1}$

(ii) $\dfrac{n!}{n^n} Z^n$. **8,8**

**5.** (a) If $\sum_0^{\infty} a_n$ converges, then $f(z) = \sum_0^{\infty} a_n z^n$ tends to $f(1)$ as $z \to 1$ in such a manner that $\dfrac{|1-z|}{(1-|z|)}$ remains bounded. **8**

(b) Find the residue at $i$ of the function :

$$\frac{1}{\log(2+iz)(z^2+1)^2}.$$ **8**

6. (a) State and prove Cauchy's Integral formula. **8**

(b) Using Cauchy's integral formula calculate :

$$\int \frac{\cosh(\pi z)dz}{z(z^2+1)}$$ on curve C, where C is the circle $|z| = 2$. **8**

## Section C

7. (a) State and prove the argument principle. **10**

(b) Prove by Argument Principle that one root of the equation $z^4 + z^3 + 1 = 0$ lies in the first quadrant. **6**

8. (a) Show that $z^4 + 2z^3 + 3z^2 + 4z + 5 = 0$ has no real or purely imaginary roots and that it has one complex root in each quadrant. **8**

(b) Show that the bounded regions determined by the closed curve are simply connected, while the unbounded region is doubly connected. 8

9. (a) Apply calculus of residue to evaluate $\int_0^{2\pi} \cos^{2n}\theta d\theta$, $n$ is a positive integer. 8

(b) Using contour integration, evaluate :

$$\int_0^{\infty} \frac{\sin mx\, dx}{x} = \frac{\pi}{2}.$$ 8

$\frac{e^{imz}}{z}$

$z = \rho e^{i\theta}$