2407716500034

Total No. of Questions - 9] [Total Pages : 3

(2115)

4644

M.A./M.Sc. (CBS) IIIrd Semester Examination

MATHEMATICS

[Analytical Number (Theory)]

(Elective-2)

Paper–M-305A

Time : Three Hours] [Maximum Marks : Regular : 80, Private : 100

Note : Attempt *five* questions in all, by selecting *one* question from each section but not more than *two* questions from any section.

SECTION–A

1. (a) Show that any integer of the form $(6k + 5)$ is also of the form $(3j + 2)$, but not conversely. 5(6)

(b) Prove that $8|(5^{2n} + 7)$ for $n \geq 1$, using mathematical induction. 5(6)

(c) Find all solutions of the following equation over $\mathbb{N}$ (set of natural numbers) : 6(8)

$$54x + 21y = 906.$$

2. (a) Prove that $(p^2 + 2)$ is composite, where p is prime and $p \geq 5$. 5(6)

(b) If p and $(p^2 + 8)$ are both primes, prove that $(p^3 + 4)$ is also prime. 5(6)

(c) A certain integer between 1 and 1200 leaves the remainders 1, 2, 6 when divided by 9, 11 and 13 respectively. Find that integer. 6(8)

3\. (a) Find all solutions of the linear congruence :

$$3x - 7y \equiv 11 \bmod 13.$$ 5(6)

(b) Determine the last 3 (three) digits of 7^{999}. 6(8)

(c) Given that $p \chi n$ for all primes $p \leq \sqrt[3]{n}$. Show that $n > 1$ is either a prime or the product of two primes. 5(6)

SECTION–B

4\. (a) If $\mathcal{F} \chi a$, prove that $\mathcal{F} | (a^3 + 1)$ or $\mathcal{F} | (a^3 - 1)$. 5(6)

(b) Prove that for any integer $n > 1$ is prime iff $(n - 2)! \equiv 1 \bmod n$. 5(6)

(c) Factor 10541 using Fermat's method. 6(8)

5\. (a) Prove that $\tau(n) \leq 2\sqrt{n}$ for any integer $n \geq 1$. 5(6)

(b) Prove that $\mu(n)\mu(n + 1)\mu(n + 2)\mu(n + 3) = 0$, for $n \in \mathbb{N}$. 5(6)

(c) Find the years from 2014-2024 when 31 December is on a Sunday. 6(8)

6\. (a) For which value of n, does $n!$ terminate in 37 zeros? 5(6)

(b) Show that if an integer n has r distinct odd prime factors, then $2^r / \phi(n)$. 5(6)

(c) If n is square-free integer, prove that $\sum_{d/n} \sigma(d^{k-1})\phi(d) = n^k$ for all integers $k \geq 2$. 6(8)

SECTION–C

7. (a) Prove that there are infinitely many primes of the form $(2kp + 1)$, where p is an odd prime. 5(6)

(b) For a prime $p > 3$, prove that the primitive roots of p occur in incongruent pairs (r, r') where $r.r' \equiv 1 \pmod{p}$. 6(8)

(c) Find $\lambda(5040)$ and $\phi(5040)$. (1+4=5)(2+4=6)

8. (a) Find the remainder when $3^{24}.5^{13}$ is divided by 17. 5(6)

(b) Solve the following equation :

$5x^2 + 6x + 1 \equiv 0 \bmod 23$. 5(6)

(c) Prove that $\sum_{a=1}^{p-2} (a(a+1)/p) = -1$, where p is an odd prime. 6(68

9. (a) Find the value of the following Legendre symbol :

$\left(\frac{1234}{4567}\right)$. 5(6)

(b) Prove that if the congruence $x^2 \equiv a \bmod 2^n$, where a is odd and $n \geq 3$, has a solution, then it has exactly four incongruent solutions. 6(8)

(c) Solve the congruence :

$x^2 \equiv 31 \bmod 11^4$. 5(6)