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Total No. of Questions - 9] [Total Pages : 3

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M.A./M.Sc. (CBS) IIIrd Semester Examination

MATHEMATICS

(Complex Analysis)

(Core)

Paper–M-301

Time : Three Hours] [Maximum Marks : Regular : 80, Private : 100

Note : Attempt *five* questions in all, selecting at least *one* question from each section but not more than *two* questions from any section. Each question have *two* subparts and each subpart carries 8 marks.

SECTION–I

1. (a) Let z_1 and z_2, be complex numbers such that $z_1 \neq z_2$ and $|z_1| = |z_2|$. If z_1 has positive real part and z_2 has negative imaginary part, then show that $\frac{z_1 + z_2}{z_1 - z_2}$ will be purely imaginary.

(b) Prove that the function $|z|^2$ is continuous everywhere but no where differentiable except at origin.

2. (a) Show that the function $u(x, y) = e^x \cos y$ is harmonic. Find its harmonic conjugate $v(x, y)$ and the corresponding analytic function $f(z) = u + iv$ in terms of z where $z = x + iy$.

(b) Illustrate the conformality of the mapping $f(z) = z^2$ on the pair of straight lines $x = c$ and $y = d$.

3. (a) Prove that

$$\log z - \log \bar{z} = 2i \tan^{-1}\left(i \cdot \frac{\bar{z} - z}{\bar{z} + z}\right).$$

(b) Prove the identity

$$\sin 3z = 3 \sin z - 4 \sin^3 z.$$

SECTION–II

4. (a) State and prove Morera's theorem.

(b) Evaluate

$$\int_C \frac{z-3}{z^2+2z+5}\, dz$$

where C is a circle given by $|z + 1 - i| = 2$.

5. (a) State and prove Weierstrass's theorem on analytic functions.

(b) What is the coefficient of z^7 in the Taylor expansion of $\tan z$ about origin?

6. (a) Find the poles and residues of the function $\dfrac{1}{\sin^2 z}$.

(b) Establish the uniqueness of Laurent's expansion.

SECTION–III

7. (a) Prove that the region obtained from a simply connected region by removing m points has the connectivity $m + 1$, and find a homology basis.

(b) Show that a single-valued analytic branch of $\sqrt{1-z^2}$ can be defined in any region such that the points ± 1 are in the same component of the complement.

8. (a) How many roots does the equation $z^7 - 2z^5 + 6z^3 - z + 1 = 0$ have in the disk $|z| < 1$?

(b) Using the calculus of residues evaluate

$$\int_0^{\frac{\pi}{2}} \frac{1}{a+\sin^2 x}\, dx$$

where $a \in \mathbb{R}$ and $|a| > 1$.

9. (a) Show that if $f(z)$ is analytic and bounded for $|z| < 1$ and if $|\zeta| < 1$, then

$$f(\zeta) = \frac{1}{\pi} \int\int_{|z|<1} \frac{f(z)}{(1-\bar{z}\zeta)^2}\, dxdy.$$

(b) Let $f(z)$ be analytic in a simply connected region Ω, the prove that for every circle $\gamma \in \Omega$,

$$\int_\gamma f(z) dz = 0.$$
